



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

**16 November 2022 (X-paper)
09:00–12:00**

Scientific calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA
NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
 2. Read all the questions carefully.
 3. Number the answers according to the numbering system used in this question paper.
 4. Start each section on a new page.
 5. Use only a black or a blue pen.
 6. Write neatly and legibly.
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QUESTION 1

1.1 Given: $Q = (x + y) \ln \left(\frac{x}{y} \right)$, show that

$$x \frac{\partial Q}{\partial x} + y \frac{\partial Q}{\partial y} = Q$$



(3)

1.2 If $z = 3x^2y^5$, find the rate of change of z when x is 3 units and y is 2 units while x is decreasing at 5 units/s and y is increasing at 2,5 units/s.

(3)

[6]**QUESTION 2**

Determine $\int y dx$ if:

2.1 $y = -2x \operatorname{cosec}^2 2x$

(2)

2.2 $y = x^3 \cos x$



(5)

2.3 $y = \frac{1}{(x+1)(x+4)}$

(4)

2.4 $y = \sin^5 3x$

(5)

2.5 $y = 5xe^{-2x}$

(2)

[18]**QUESTION 3**

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{x^3 + 6x - 7}{(x^2 - 2x - 3)^2} dx$



(6)

3.2 $\int \frac{x^2}{(1-x)(x^2+1)} dx$

(6)

[12]

QUESTION 4

- 4.1 Determine the general solution of:

$$(t-1)^3 \frac{ds}{dt} + 4(t-1)^2 s = t+1$$



(5)

- 4.2 Determine the particular solution of:

$$\frac{d^2s}{dt^2} + 4\frac{ds}{dt} + 13s = 39t^2 + 11t + 54 \text{ when } t=0; s=1 \text{ and } \frac{ds}{dt}=8 \text{ when } t=0.$$

(7)
[12]**QUESTION 5**

- 5.1 5.1.1 Make a neat sketch of graph $y = \sin 2x$. Show the representative strip/element that you will use to calculate the volume if the area bounded by the graph and the ordinates $y = 0$ and $x = \frac{\pi}{2}$ rotate about the x -axis. (2)

- 5.1.2 Calculate the volume generated if the area described in QUESTION 5.1.1 rotates about the x -axis. (3)

- 5.1.3 Calculate the volume moment about the y -axis as well as the distance of the centre of gravity from the y -axis. (5)



- 5.2 5.2.1 Calculate the points of intersection of the two curves $y = 2x$ and $y = x^2$. Make a neat sketch of the two curves and show the area bounded by the curves. Show the representative strip/element PARALLEL to the x -axis that you will use to calculate the area bounded by the curves. (3)

- 5.2.2 Calculate the area bounded by the two curves as described in QUESTION 5.2.1. (3)

- 5.2.3 Calculate the second moment of area when the area bounded by the two curves as described in QUESTION 5.2.1 is rotated about the x -axis. (4)

- 5.2.4 Express the answer in QUESTION 5.2.3 in terms of the area. (1)

- 5.3 5.3.1 Make a neat sketch of the graph $y = -x^2 + x$ and show the representative strip/element that you will use to calculate the volume of the solid generated when the area bounded by the curve and the line $y = 0$ rotates about the x -axis. (2)



- 5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

- 5.3.3 Calculate the moment of inertia of the solid obtained when the area described in QUESTION 5.3.1 rotates about the x -axis. (4)

- 5.4 5.4.1 A vertical sluice gate in the shape of a parabola is placed in a dam wall. The top of the gate is 8 m wide and lies 1 m below the water level. The sluice gate is 4 m high.



Make a neat sketch of the sluice gate and show the representative strip/element that you will use to calculate the first and the second moment of area of the gate about the water level.

Calculate the relation between the two variables x and y . (3)

- 5.4.2 Calculate, by using integration, the area moment of the retaining wall about the water level in QUESTION 5.4.1. (3)

- 5.4.3 Calculate, by using integration, the second moment of area of the retaining wall described in QUESTION 5.4.1 about the water level as well as the depth of the centre of pressure on the retaining wall. (4)

[40]



QUESTION 6

- 6.1 Calculate the length of the curves $x = 3 \cos t - \cos 3t$ and $y = 3 \sin t - \sin 3t$ for $0 \leq t \leq \pi$. (6)

- 6.2 Calculate the surface area generated when the curve given by $y = \frac{1}{2}(e^x + e^{-x})$ from $x = 0$ to $x = 2$ rotates about the x -axis. (6)

[12]



TOTAL: 100

MATHEMATICS N6**FORMULA SHEET**

Any other applicable formula may also be used.

Trigonometry

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$$

$$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$$

$$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$$

$$\tan x = \frac{\sin x}{\cos x}; \quad \sin x = \frac{1}{\operatorname{cosec} x}; \quad \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n dx$
e^{ax+b}	$e^{ax+b} \cdot \frac{d}{dx} (ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx} (ax+b)} + C$
a^{dx+e}	$a^{dx+e} \cdot \ln a \cdot \frac{d}{dx} (dx+e)$	$\frac{a^{dx+e}}{\ln a \cdot \frac{d}{dx} (dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} \cdot \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} \cdot \ln a \cdot \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} \cdot \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x) dx$
$\sin f(x)$	$\cos f(x) \cdot f'(x)$	-
$\cos f(x)$	$-\sin f(x) \cdot f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \cdot f'(x)$	-
$\cot f(x)$	$-\operatorname{cosec}^2 f(x) \cdot f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \cdot f'(x)$	-
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x) \cot f(x) \cdot f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2 + 1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2 + 1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x) \sqrt{[f(x)]^2 - 1}}$	-
$\operatorname{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x) \sqrt{[f(x)]^2 - 1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a} \tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x) dx$
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$$\cot^2(ax) \quad - \quad -\frac{1}{a} \cot(ax) - x + C$$

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

$$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2 x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2 x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2 x^2} dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2} \sqrt{a^2 - b^2 x^2} + C$$

$$\int \frac{dx}{a^2 - b^2 x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$$

$$\int \sqrt{x^2 \pm b^2} dx = \frac{x}{2} \sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$$

$$\int \frac{dx}{\sqrt{b^2 x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2 x^2 \pm a^2} \right] + C$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y dx; A_x = \int_a^b (y_1 - y_2) dx$$

$$A_y = \int_a^b x dy; A_y = \int_a^b (x_1 - x_2) dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx; V_x = \pi \int_a^b (y_1^2 - y_2^2) dx; V_x = 2\pi \int_a^b xy dy$$

$$V_y = \pi \int_a^b x^2 dy; V_y = \pi \int_a^b (x_1^2 - x_2^2) dy; V_y = 2\pi \int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = \int_a^b r dA \quad A_{m-y} = \int_a^b r dA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b r dA}{A}; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b r dA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b r dV \quad ; \quad V_{m-y} = \int_a^b r dV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b r dV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b r dV}{V}$$

MOMENTS OF INERTIA

Mass = Density $\times$ volume

$$M = \rho V$$

DEFINITION: $I = m r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \quad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots + \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3 (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$A_x = \int_{u_1}^{u_2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2} du$$

$$A_y = \int_{u_1}^{u_2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2} du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$S = \int_{u_1}^{u_2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2} du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore \quad ye^{\int P dx} = \int Qe^{\int P dx} dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A \cos bx + B \sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) \frac{d\theta}{dx}$$